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Modelling Stability of Growth Between Mathematics and Science Achievement via Multilevel Designs with Latent Variables

by

Lingling Ma

A thesis submitted to the Faculty of Graduate Studies and Research in partial fulfillment of the requirements for the degree of

Master of Science

in

Statistics

Department of Mathematical and Statistical Sciences

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Faculty of Graduate Studies and Research

The undersigned certify that they have read, and recommend to the Faculty of Graduate Studies and Research for acceptance, a thesis entitled **Modelling Stability of Growth Between Mathematics and Science Achievement via Multilevel Designs with Latent Variables** submitted by **Lingling Ma** in partial fulfillment of the requirements for the degree of Master of Science in Statistics.

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Dr. Xin Ma (Co-Supervisor)

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July 29, 2003

Abstract

In this thesis, a series of multilevel models was introduced to handle hierarchically structured data. A multivariate multilevel model was adopted for multivariate response data; a multilevel model with latent variables was utilized for the data with measurement errors; a multilevel growth model was applied to repeated measures data. Based on these models, we derived a multivariate multilevel growth model with latent variables to model the consistency among students and among schools in the rates of growth for "true" mathematics achievement and "true" science achievement after adjusting for measurement errors during the entire middle and high school years, with data from the Longitudinal Study of American Youth (LSAY). There was no evident consistency of growth among students, and the inconsistency was not much influenced by student characteristics and school characteristics. However, there was evident consistency among schools, and this consistency was influenced by student characteristics and school characteristics. The relationship between the rates of growth for "true" mathematics achievement and "true" science achievement was much more evident at the school level than at the student level.

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List of Acronyms

ACKPRS	–	Academic Press
ALG	–	Algebra
ANCOVA	–	Analysis of Covariance
ANOVA	–	Analysis of Variance
BAS	–	Basic Mathematics Skills
BIO	–	Biology
COMPMATH	–	Computer-student Ratio in Mathematics
DISCLM	–	Disciplinary Climate
EDLEVEL	–	Teacher's Education Level
ENV	–	Environmental Sciences
EXPERIEN	–	Teaching Experience in Mathematics
EXTRACUM	–	Extracurricular Activities
FATHSEI	–	Father's Socioeconomic Index Scores
FREELUN	–	Percentage of Free-lunch Students
GEO	–	Geometry
GLS	–	Generized Least Squares
GRASPAN	–	Grade Span
GREASUPT	–	General Support for Mathematics
HOMEWORK	–	Math Homework
HLM	–	Hierarchy Linear Model
IGLS	–	Iterative Generized Least Squares
IRT	–	Item Response Theory
LSAY	–	Longitudinal Study of American Youth

MOTHSEI	–	Mother's Socioeconomic Index Scores
NUMPAR	–	Number of Parents
OLS	–	Ordinary Least Squares
PHY	–	Physics
PRILED	–	Principal Leadership
PROMINO	–	Proportion of Minornity Student
QLT	–	Quantitative Literacy
RIGLS	–	Restricted Iterative Generalized Least Squares
SCHSIZE	–	School Enrollment Size
SES	–	Social Economy Status
STFCOP	–	Staff Cooperation
STRATIO	–	Teacher-student Ratio
TCHATU	–	Teacher Autonomy
TCHCOM	–	Teacher Commitment
VWS	–	Overall Parental Involvement

Chapter 1

Introduction

Most studies of school effects focus on only a single school subject (e.g. mathematics) as an academic outcome (Sammons et al., 1996). "A relatively neglected area of school effectiveness research is the extent of consistency in schools' effects on different outcomes at the same point in time" (Thomas et al., 1997, p. 169). Even rarer are studies of school effects on the consistency of growth in different academic outcomes across multiple points in time. It highlights the need for further detailed longitudinal research to address the issues of stability and consistency.

The present study was an attempt to fill these gaps. We examined the growth in the "true" mathematics achievement and the growth in the "true" sciences achievement, attempting to discover whether students who improve quickly in mathematics achievement also do so in science achievement and what student and school characteristics influence the consistency (or inconsistency). We also examined this issue from the school's perspective. In particular, two principal research questions were investigated:

1. Do students (schools) have consistent rates of growth across the "true" mathematics and the "true" science achievement? That is, if students (schools) grow faster in one area, do they also grow fast in the other area?

2. After adjusting for student and school characteristics, do students (schools) have a consistent growth trajectory across mathematics and science areas? That is, how do student and school characteristics affect the correlation in rates of growth between the "true" mathematics and "true" science achievement?

It is appropriate to study mathematics and science learning together, since mathematics and science are often spoken of together as if the two were synonymous (Gibson, 2001), and they also often share similar ways of learning and teaching and similar methods of logical thinking and problem solving (Ma, 2000). In general, with control over student characteristics, correlations among students between mathematics achievement and science achievement were reported as 0.35 and 0.7 at the secondary school level and elementary level, respectively (Sammons et al., 1997; Thomas et al., 1997).

Luyten (1994) studied the consistency of secondary school effects both across school subjects and across academic years, as well as the interaction between subjects and years. School effects during a certain academic year were fairly consistent across academic years, but school effects in a certain year were remarkably different across school subjects. More importantly, Luyten (1994) reported that a substantial interaction existed between subjects and years, so that a school could achieve outstanding outcomes in a certain school subject one year but could produce poor outcomes in the same school subject the next year. These findings lead to the conclusion that school departments

are differentially effective in their teaching.

Ma (2001) examined the correlation between four academic outcomes (mathematics, science, reading, and writing) among both students and schools. Without adjustment for student and school characteristics, correlation between mathematics and science was 0.58 among students and 0.70 among schools. With adjustment for student characteristics, correlation between mathematics and science was 0.57 among students and 0.67 among schools. With adjustment for both student and school characteristics, correlation between mathematics and science was 0.57 among students and 0.65 among schools. Considering these correlation coefficients weak, Ma (2001) concluded that both students and schools were differentially successful in learning and teaching mathematics and science.

Willms and Raudenbush (1989) pointed out another chief limitation in previous research on school effects – the lack of adequate statistical control over school characteristics. They contended that many previous analyses of school effects were done without any understanding of school processes at the school level. School characteristics can often be classified into two sets of variables. One set describes the context of a school, including school enrollment size, school location, and percentage of free-lunch students. The other set of school-level variables, often referred to as "evaluative variables," are associated with the climate of the school. The evaluative variables attempt to portray the inner working of school life, for example, the organization of the school, the school operation, and the attitudes, values, and expectations of students, parents, and teachers. Studies of school effects on the consistency of growth in different school subjects should examine the influence of school characteristics on this

consistency.

The data for the present study came from the second cohort of the stratified national probability sample of 52 schools in the Longitudinal Study of American Youth (LSAY). Beginning in the fall of 1987, the LSAY was a longitudinal panel study of public middle and high school students. About 60 seventh graders were randomly selected in each of the 52 schools, and the sample size was 3116 students. They were followed for six years from Grade 7 to Grade 12 (Miller et al., 1999). With a focus on mathematics and science education, the information from students, parents, and teachers was also included in the study (Miller & Hoffer, 1994).

The difference in student academic achievement could be explained by student characteristics including both individual and family characteristics. For this reason, gender, age, race-ethnicity, mother's socioeconomic status (SES), father's SES, number of parents, and number of siblings were used in this study as explanatory variables at the student level. Considering the above review, not only school context variables but also school climate variables were used in our study.

In educational effectiveness research, multilevel models have become popular since these models take account of the hierarchical structure of the data (Opdenakker et al., 2000). In the social sciences, hierarchical structured data arise routinely where the students are nested within the schools (Hox, 1995). The multilevel structure could not be ignored without potentially invalidating many of the traditional statistical analysis techniques (Goldstein, 1995). Multilevel modelling was used as the method of analysis In its detail, this is a relatively new approach to the analysis of hierarchically structured data but

in broad terms is a refined version of a standard and well-proved method of statistical analysis, multiple regression. The advantage of multilevel modelling is that it exploits the hierarchical structure of the data, and also, like multiple regression, it can be used to look at potentially interesting differences, such as those between the performance of males and females (Thomas et al., 1997).

Previously, a multivariate multilevel model was developed for the multivariate response data; for the data with measurement errors, a multilevel model with latent variables was adopted, and a multilevel growth model was built for repeated measures data. Based on all these models, we derived a multivariate multilevel growth model with latent variables to model the consistency among students and among schools in the rates of growth among the "true" mathematics achievement and the "true" science achievement. We represented measurement errors as a level within the hierarchical model. At level 1, a design matrix was built to link the items with the "true" scores (or latent variables). At level 2, the latent variables varied among grades within students grouped in schools. And at level 3 and levels 4, the coefficients from the level-2 model varied randomly across students and schools, respectively. Therefore, the level-1 model was a measurement model and the level-2 model was a growth model, whereas the next two levels represented a normal hierarchical linear model.

The structure of this thesis is as follows. Chapter 2 describes the data sources used for our analysis. Chapter 3 reviews the multilevel models related to educational effective research and generalizes the problem. A multivariate multilevel model with latent variables for repeated measures data is derived in detail. Meanwhile, the related parameter estimation is reviewed. Chapter 4

and Chapter 5 present data analysis and principle findings from the multilevel model, respectively. Finally, Chapter 6 summarizes the model design, discusses the findings from the analysis, and points out the directions for further research on this topic.

Chapter 2

Data

The data for the present study came from the second cohort of the stratified national probability sample of the schools involved in the LSAY (Miller et al., 1999). Beginning in the fall of 1987, the LSAY was a longitudinal panel study of public middle and high school students. With a focus on mathematics and science education, about 60 seventh graders were randomly selected in each of the 52 schools, and the total sample size was 3116 students. These students were followed for six years from Grade 7 to Grade 12, writing mathematics and science achievement tests and completing student questionnaires annually. The information from students, parents, and teachers was also included in the study (Miller & Hoffer, 1994).

Outcome Measurement

There were two sets of variables at the student level: schooling outcome variables and student background variables. The outcome measurements were seven variables describing two subject areas: mathematics and science. Basic mathematics skills (BAS), algebra (ALG), geometry (GEO), quantitative liter-

acy (QLT), biology (BIO), physics (PHY), and environmental sciences (ENV) were the seven item scores.

The BAS subscale measured the achievement in the basic mathematics skills by using the items on the mathematics achievement test calibrated to measure an understanding of basic mathematics skills, and so on. Item scores in each subject were imputed scores including non-aberrant observed scores when available. These items were stored as continuous variables in the data file. These scores were comparable across grade levels within each school subject. The available data for each subject in each year are listed in Table 2.1. Some data were missing because some students were absent during the testing. Nevertheless, all available student achievement scores were used as dependent measures.

Table 2.1: Valid data for seven subjects in six years

	BAS	ALG	GEO	QLT	BIO	PHY	ENV
Grade 7	3071	3071	3071	3065	3065	3065	3065
Grade 8	2527	2527	2527	2581	2581	2581	2581
Grade 9	2326	2326	2326	2241	2241	2241	2241
Grade 10	1980	1980	1980	2040	2040	2040	2040
Grade 11	1588	1588	1588	1593	1593	1593	1593
Grade 12	1145	1145	1145	1168	1168	1168	1168

The first four item scores (BAS, ALG, GEO and QLT) are the measures of the latent true score for mathematics achievement with measurement errors, respectively (Muthen, 1996). The last three item scores (BIO, PHY and ENV)

are the measures of the corresponding latent true score for science achievement with measurement errors. Therefore, a measurement model that links the items and the latent true scores for each person was necessary (Raudenbush et al., 1991).

Student-level Variables

Student characteristics can be individual characteristics, such as gender and age, or family characteristics, such as number of parents and socioeconomic status (Ma & Klinger, 2000). Gender difference, age, race and ethnicity, and socioeconomic gaps have long been used to explain the differences in student academic achievement (Thomas et al., 1997; Ma, 1999; Ma, 2000). Family structure (number of parents and number of siblings) also effects academic achievement (Ma & Klinger, 2000). In this way, gender, age, race-ethnicity, mother's SES, father's SES, number of parents, and number of siblings were selected in this study as explanatory variables at the student level.

Gender came from student self-reports obtained in the fall of 1987. When it was missing, the value of gender was assigned by inspection of the students' first name or by checking the school rosters. If the value was contradicted by the student's given name, it was replaced. Female was recoded as a dichotomous variable comparing females with males, with 0=male and 1=female. The month of birth and year of birth were asked in the LSAY. Based on them, age was calculated as the number of months since birth.

The LSAY identified the ethnic background of students by six categories: (a) Hispanic, (b) Black, (c) White, (d) Asian, (e) Native American, and (f) others. We concentrated our analysis on five ethnic groups - Hispanic, Black,

White, Asian and Others, which included Native American. Four dummy variables were created to represent race-ethnicity with White as the baseline category against which Hispanic, Black, Asian, and Others were compared.

As main explanatory variables for a student's social background, mother's SES and father's SES were standardized composite variables constructed based on parents' self-reported education and occupational status, as well as student-reported household possessions (Miller & Hoffer, 1994; Ma & Willms, 1999).

Marital status and the number of children were obtained from the parent interviews. There were five categories of marital status: married, widowed, divorced, separated, and never married. One dummy variable was created to represent the number of parents from marital status, with married as 1 and other categories as 0, comparing both-parent families with single-parent families based on the base-year (1987 -1988) data. Number of siblings was created based on the number of children, which is a continuous variable. All the variables were examined for extreme data, with corrections or deletions.

School-level Variables

At the school level, there are two sets of variables. One set describes the context of a school, including school location and percentage of free-lunch students. The other set of school-level variables depicts school climate, such as principal leadership, disciplinary climate, academic press, and parental involvement (Ma & Klinger, 2000).

School enrollment size, school location, and percentage of free-lunch students were used as our independent school context variables. School location had three categories: urban school, suburban school, and rural school. Dummy

recoding of school location created two variables with urban school as the baseline category against which suburban school and rural school were compared. The percentage of students eligible for federal lunch assistance was used to measure a school's socioeconomic composition.

According to our previous research on the consistency in the rates of growth between mathematics achievement and science achievement without adjusting for measurement errors within student academic achievement, school-level variables associated with parental involvement did not show any significant impacts on the consistency among students and schools. Therefore, not many variables related to parental involvement, such as home discussion, home supervision, home-school communication, and volunteering work were included in our present research. Only one variable VWS indicating overall parental involvement was included.

At the school level, our independent variables included measures of (a) principal leadership, (b) academic press, (c) disciplinary climate, (d) teacher autonomy, (e) teacher commitment, and (f) general support for mathematics. These were derived from data for 42 items on the teacher questionnaire, and were coded such that high values indicated a more positive school climate.

Selected items for school-level variables:

(a) Principal leadership

BE 17 The principal deals effectively with pressures from outside the school that might interfere with my teaching.

BE 18 The principal sets priorities, make plans, and sees that they are carried out.

BE 19 Staff are involved in making decisions that affect them.

BE 20 The school administration's behavior toward the staff is supportive and encouraging.

BE 22 The principal seldom consults with staff members before he/she makes decisions that affect us.

BE 23 Goals and priorities for the school are clear.

BE 27 The principal lets staff know what is expected of them.

(b) Academic press

BE 16 The learning environment in this school is not conducive to school.

(1=disagree strongly, 2=disagree somewhat, 3=disagree, 4=agree, 5=agree somewhat, 6=agree strongly)

Achievement for most students

BE 25 Staff members maintain high standards of performance.

BE 30 Most students in this school work up to their ability.

BE 31 The teachers in this school push the students pretty hard in their academic subjects.

BE 32 In this school, there is really very little a teacher can do to insure that all of his/her students achieve at a high level.

(1=disagree strongly, 2=disagree somewhat, 3=disagree, 4=agree, 5=agree somewhat, 6=agree strongly)

(c) Disciplinary climate

To what extent is each of the following a problem in your school?

BE 48 Students absenteeism from class

BE 52 Robbery, theft, or vandalism

BE 53 Student use of drugs or alcohol

BE 54 Verbal abuse of teachers

(1=serious, 2=moderate, 3=minor, 4=none)

(d) Teacher autonomy

How much influence do teachers in your school have over policy in each of the areas bellow?

BE 39 Determining student behavior codes

BE 40 Determining the content of inservice programs

BE 41 Setting policy on grouping students in classes by ability

BE 42 Establishing the school curriculum

(1=none, 2=minor, 3=some, 4=moderate, 5=much, 6=a great deal)

How much control do you feel you have in your classroom over each of the following areas of your planning and teaching?

BE 43 Selecting textbooks and other instructional materials

BE 44 Selecting content, topics, and skills to be taught

BE 45 Selecting teaching techniques

BE 46 Discipline students

BE 47 Determining the amount of homework to be assigned

(1=none, 2=minor, 3=some, 4=moderate, 5=much, 6=a great deal)

(e) Teacher commitment

BE 24 There is a great deal of cooperative effort among staff.

BE 26 I usually look forward to each working day at this school.

BE 28 I sometimes feel it is a waste of time to try to do my best as a teacher.

BE 29 I am familiar with the content and specific goals of the courses taught by other teachers in my department.

(1=disagree strongly, 2=disagree somewhat, 3=disagree, 4=agree, 5=agree somewhat, 6=agree strongly)

To what extent is each of the following a problem in your school?

BE 50 Teachers' lack of commitment or motivation

BE 51 Teacher morale

(1=serious, 2=moderate, 3=minor, 4=none)

(f) General support for mathematics

For each factor, please indicate the extent to which it is a problem in your school.

BE 77 Some teachers are inadequately trained to teach mathematics

BE 78 Lack of teacher planning time

BE 79 Class sizes too large

BE 80 Students with different abilities and interests taking the same mathematics classes

BE 81 Too little coordination or articulation between classes in the mathematics curriculum

BE 82 Too few advanced mathematics courses in the curriculum

(1=serious, 2=moderate, 3=minor, 4=none)

Other school climate variables were included: teaching experience in mathematics (EXPERIEN), teacher's education level (EDLEVEL), staff cooperation (STFCOP), math homework (HOMEWORK), grade span (GRASPAN), teacher-student ratio (STRATIO), proportion of minority student (PROMINO), computer-student ratio in mathematics (COMPMATH), and extracurricular activities (EXTRACUM).

Chapter 3

Multilevel Analysis

3.1 Hierarchical data structure

Many kinds of data in the social sciences have a multilevel or hierarchical structure (Goldstein, 1995; Rasbash et al., 2000; Raudenbush & Bryk, 2002). School education systems provide an obvious example of a hierarchical structure, with students grouped in classes, which themselves are nested within schools. Once groupings are established, individuals are subject to the influence of grouping; thus, responses from the individuals are correlated. A 3-level hierarchy is therefore created by the school system: students are the level 1 units clustered within classes, which are the level 2 units, and the schools are the level 3 units (Kreft, 1996).

In student growth studies, an example of hierarchical structured data occurs when repeated measurements over time are taken from individuals, who are in turn grouped within schools. Such structures are typically strong hierarchies since the variation within students is much smaller than the one between

students. Here the repeated measurements constitute the level 1 units, with students representing level 2 units in a 3-level structure where the level 3 units are schools.

Another special case of multilevel data is that several measurements are taken of individuals, who are clustered within schools. The multilevel structure is analogous to that for repeated measures data, with the measurements taking the place of occasions.

The existence of such data hierarchies is neither accidental nor ignorable. Failure to consider the hierarchical nature of this data leads to unreliable estimation of the effectiveness of school policies and practices (Goldstein, 1995; Raudenbush & Bryk, 2002). Once the groupings are established, the group and group members both influence and are influenced by the group membership. In all instances mentioned above, the responses are no longer independent of each other. This factor may invalidate many of the traditional statistical analysis techniques, which assume the independence of the responses (Boyle & Willms, 2001). Multilevel modelling is developed specifically to account for correlated response variables at multiple levels; hence, it solves the dilemmas in the analysis of hierarchical data (Goldstein, 1995; Rasbash et al., 2000; Raudenbush & Bryk, 2002).

3.2 Review of multilevel models

Over the last decade, hierarchical linear models, or multilevel models, have become popular for dealing with hierarchical data and providing an efficient and valid approach (Goldstein, 1995; Rasbash et al., 2000; Raudenbush &

Bryk, 2002). In its detail, it is a relatively new approach to the analysis of hierarchically structured data but in broad terms is a refined version of a standard and well-proved method of statistical analysis, multiple regressions. The advantage of multilevel modelling is that it exploits the hierarchical structure of the data, and also, like multiple regression, it can be used to look at potentially interesting differences, such as those between the performance of males and females (Thomas et al., 1997).

In hierarchical models, the assumption of independence of observation is dropped, and the relationships among the data are no longer assumed to be fixed over contexts. Based on more realistic assumptions such as random coefficient and intra-class correlation, hierarchical modelling results in more efficient estimates and more powerful tests than the traditional regression model (Goldstein, 1995).

In Ma (2001), the advantages of multilevel modelling are reviewed as follows: Multilevel modelling is a significant step further in the solution of the persistent problems in the analysis of hierarchical data, accounting for (a) the dilemma of unit of analysis (Bryk & Raudenbush, 1992), (b) the problem of dependencies of individual responses within groups (clustering effects) (Hox, 1995), (c) confounding variables at both within and between group levels (Kreft & de Leeuw, 1998), (d) cross-level interactions (Raudenbush & Willms, 1991), and (e) the manipulation of random coefficients (Bryk & Raudenbush, 1992).

The subsections below will give a detailed discussion of multilevel level models. Subsection 3.2.1 will introduce a 2-level hierarchical model and its applications to school effectiveness. The basic structure and applications of

the multivariate multilevel model are reviewed in Subsection 3.2.2. Furthermore, the use of the multivariate multilevel model to deal with latent variables is discussed in Subsection 3.2.3. Later, the multilevel model for repeated measures data is given in the last subsection,. Now we begin with the simplest multilevel model.

3.2.1 The 2-level model and basic notation

The simplest possible hierarchical linear model is equivalent to a one-way ANOVA with random effects (Raudenbush & Bryk, 2002).

The level-1 model is

$$Y_{ij} = \alpha_{0j} + e_{ij}, \tag{3.1}$$

where error term e_{ij} is the level-1 random effects, which is assumed to have normal distribution with a mean of 0 and a constant level-1 variance σ^2. α_{0j} is the mean outcome for the jth unit.

The level-2 model for the one-way ANOVA with random effects is

$$\alpha_{0j} = \beta_{00} + u_{0j}, \tag{3.2}$$

where β_{00} represents the grand-mean outcome in the population, and u_{0j} is the random effect associated with unit j and is assumed to have a mean of zero and variance T_α. The fundamental idea underlying multilevel modelling is that the parameters from one level become the outcome or response variables at the next level (Boyle & Willms, 2001).

The combined model based on (1) and (2) is

$$Y_{ij} = \beta_{00} + u_{0j} + e_{ij}, \tag{3.3}$$

which is a one-way ANOVA model with a grand mean β_{00}, a group (level-2) effect, u_{0j}, and with a person (level-1) effect, e_{ij}.

$$Var(Y_{ij}) = Var(u_{0j} + e_{ij}) = T_a + \sigma^2 \tag{3.4}$$

The σ^2 represents the within-group variability, and T_a captures the between-group variability.

More complex hierarchical models were introduced by Raudenbush (2002), including a regression model with means-as-outcomes, a one-way analysis of covariance model with random effects, a random-coefficients regression model, a model with intercepts- and slopes-as-outcomes, and a model with nonrandomly varying slopes.

A two-level HLM was applied to the data from the New Brunswick School Climate Study (Ma & Klinger, 2000) to estimate the effects of student and school variables on academic achievement at student and school levels.

The two-level HLM is as follows:

$$Y_{ij} = \alpha_{0j} + \alpha_{1j}X_{ij} + e_{ij}, \tag{3.5}$$

where

$$\alpha_{0j} = \beta_{00} + \beta_{10}Z_j + u_{0j}. \tag{3.6}$$

The combined model is as follows:

$$Y_{ij} = \beta_{00} + \beta_{10}Z_j + \alpha_{1j}X_{ij} + u_{0j} + e_{ij}, \tag{3.7}$$

where Y_{ij} was the student academic achievement, X_{ij} was the student-level variables, and the student-level residual term e_{ij} was assumed to follow a normal distribution with a mean of zero and variance σ^2. α_{0j} represented the

mean outcome for jth school after controlling all the significant student-level variables. α_{1j} described the relationship between the academic achievement and student-level variables after adjusting all other variables. Z_j was the school-level variables, and the school-level error term u_{0j} was assumed to follow a normal distribution with a mean of zero and variance T_α. β_{00} represented the grand-mean outcome in the schools after adjusting all school-level variables, and β_{10} was the school-level coefficients describing the relationship between the adjusted mean outcome and school-level variables.

The two-level HLM analysis can be obtained through three stages. At the first stage, the analysis produces the null model with no student-level and school-level variables. Similar to the random-effect ANOVA model, the null model provides a measure of the variances within and between schools. At the second stage, the student variables are added to the null model separately. Later, only the significant variables are retained to determine which one has a significant effect on the academic measures in the presence of other variables. Those student-level variables, which have a significant relative effect on the academic measures, are kept in the final model. A similar procedure is applied to the school-level variables at the third stage. With the use of a HLM, the variation in achievement is explored. In addition, the relative and absolute effects for students and schools variables are determined.

3.2.2 Multivariate multilevel model

A special case of a multilevel model is the multivariate multilevel model. Several measurements, for example, mathematics and sciences scores, can be taken of an individual. If these measurements are analyzed together as response vari-

ables, we should use a multivariate multilevel model, where the measurements of individuals constitute level 1 units, and each individual is considered as a level 2 unit.

The basic explanatory variables are a set of dummy variables indicating which response variable is present. Further explanatory variables are defined as multiplying these dummy variables by individual level explanatory variables. The level-one model is used only to build the multilevel structure; hence, there is no level 1 variation (Goldstein, 1995).

$$Y_{ij} = \alpha_{1j} X_{1ij} + \alpha_{2j} X_{2ij}, \tag{3.8}$$

where

$$X_{1ij} = \begin{cases} 1, & \text{if } i = \text{math}, \\ 0, & \text{if } i = \text{scie} \end{cases}$$

and

$$X_{2ij} = \begin{cases} 0, & \text{if } i = \text{math}, \\ 1, & \text{if } i = \text{scie}. \end{cases}$$

The second level is the individual level where each coefficient from the individual model is modelled through individual explanatory variables.

$$\begin{aligned} \alpha_{1j} &= \beta_{10} + \beta_{1j} Z_j + u_{1j} \\ \alpha_{2j} &= \beta_{20} + \beta_{2j} Z_j + u_{2j}, \end{aligned} \tag{3.9}$$

where z_j is the individual level background variables, and $Var(u_{1j}) = \sigma^2_{u1}$, $Var(u_{2j}) = \sigma^2_{u2}$, and $Cov(u_{1j}, u_{2j}) = \sigma_{u12}$. The level 2 variances and covariance are the between-student variances. Multilevel models explore the relationship between the measurements at both the school and student level and analyze whether any individual background variables have significant effects on the relationship.

In Ma (2001), a three-level multivariate multilevel model was developed to explore the stability of school academic performance across four subject areas. The first level was the within-student model with four dichotomous variables denoting each of the four subject areas: mathematics, science, reading, and writing.

$$Y_{ijk} = \alpha_{1jk}X_{1ijk} + \alpha_{2jk}X_{2ijk} + \alpha_{3jk}X_{3ijk} + \alpha_{4jk}X_{4ijk}, \tag{3.10}$$

where

$$X_{1ijk} = \begin{cases} 1, & \text{if } i = \text{math}, \\ 0, & \text{if } i \neq \text{math}, \end{cases}$$

and so on for all other $X_{.ijk}$. Y_{ijk} was the outcome score in subject area i for student j in school k. This measurement model with dichotomous variables served as a device to combine the outcomes of four subject areas into one equation, thus allowing the removal of the intercept and the error at the first level and, moreover, allowing the calculation of the matrices of variance and covariance among the four subject areas at the higher levels of the analysis. In other words, these matrices could not be obtained with the univariate approach of analysis (four separate HLM models). Also, because this within-student model was only a measurement device (the model has no error terms), Y_{ijk} was not the predicted value as normally assumed in regression analysis but the actual score in subject area i for student j in school k (Ma, 2000).

The second level was the student model where each coefficient from the within student model was modelled through student background characteristics. For instance, the coefficient for mathematics was explained as student level variables.

$$\alpha_{1jk} = \beta_{10k} + \beta_{1jk}Z_{jk} + u_{1jk}, \tag{3.11}$$

where α_{1jk} is the mathematics achievement, β_{10k} is a measure of average performance for school k adjusted for all student-level significant variables, and student-level coefficient β_{1jk} measures the relationship between mathematics achievement and student-level variable Z_{jk}.

The third level was the school model where constants from the student model were modelled through school characteristics.

$$\beta_{10k} = \beta_{100} + \beta_{101} W_k + v_{10k} \tag{3.12}$$

Based on the multilevel model, the correlations of academic performance across the four subjects among students and among schools were examined, also with statistical adjustment for student characteristics and school context and climate characteristics (Ma 2001).

In school-effects research, all student-level coefficients should be fixed at the school level, except those that are objects of investigation (Ma, 2001). Since in the model above, all student-level coefficients were fixed, the relationship between student-background characteristics and academic achievement measures could not be examined. However, one could set any student-level coefficient instead of the intercept from the second level model as the response variable in the third level model to explore the relationship between student-background characteristics and academic achievement measures. Ma (2000) also built a multivariate multilevel model to investigate the relationship between SES and academic achievement measures, exploring some equity effects across academic achievement measures. In this case, the variance and covariance among slopes were examined.

Nevertheless, test scores in each subject area were assumed to be measured without errors in this analysis; hence, this multilevel model did not remove

measurement errors from the analysis of correlates at the student level (Ma, 2001). In the next subsection, a model for measurement errors within students' outcomes will be introduced.

3.2.3 Multivariate multilevel model with latent variables

A multilevel multivariate model can also be applied to the latent variables (Raudenbush et al., 1991). A three-level hierarchical model was utilized by Raudenbush (1991) to analyze the organizational structure and climate. The first level of the model was a measurement model describing the linkage between the items and the latent true scores for each person and parameterizing the measurement error variation. At the second level, the true scores were the outcomes to be predicted by teacher-level characteristics. The parameters of this teacher-level model can vary randomly from school to school. At the third level, a model was formulated for the variation and covariation of these school-level parameters. Thus, the highest two levels of the model may be viewed as a multivariate two-level model for the latent true scores, with the lowest level of the model serving as a measurement model.

Measurement model is as follows:

$$\begin{pmatrix} Y_{1jk} \\ Y_{2jk} \\ Y_{3jk} \\ Y_{4jk} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \alpha_{1jk} \\ \alpha_{2jk} \end{pmatrix} + \begin{pmatrix} e_{1jk} \\ e_{2jk} \\ e_{3jk} \\ e_{4jk} \end{pmatrix} \tag{3.13}$$

The first two item scores, Y_{1jk} and Y_{2jk}, were measures of the latent true score α_{1jk} with measurement errors e_{1jk},e_{2jk}. The second pair of scores, Y_{3jk} and

Y_{4jk}, were measures of the latent true score α_{2jk} with measurement errors e_{1jk}, e_{2jk}, respectively.

Teacher-level model:

$$\begin{aligned} \alpha_{1jk} &= \beta_{10k} + \beta_{11k} Z_{jk} + u_{1jk} \\ \alpha_{2jk} &= \beta_{20k} + \beta_{21k} Z_{jk} + u_{2jk} \end{aligned} \tag{3.14}$$

$\beta_{..k}$ was the regression coefficients governing the relationship in school j between the latent outcome $\alpha_{.jk}$ and the teacher-level variables Z_{jk}. The random vector $\begin{pmatrix} u_{1jk} \\ u_{2jk} \end{pmatrix}$ followed a normal distribution with mean vector 0 and variance-covariance matrix T_α.

School-level model:

$$\begin{aligned} \beta_{10k} &= \gamma_{100} + \gamma_{101} W_k + v_{10k} \\ \beta_{11k} &= \gamma_{110} + \gamma_{111} W_k + v_{11k} \\ \beta_{20k} &= \gamma_{200} + \gamma_{201} W_k + v_{20k} \\ \beta_{21k} &= \gamma_{210} + \gamma_{211} W_k + v_{21k} \end{aligned} \tag{3.15}$$

At the school level, the random coefficients $\beta_{..k}$ varied as a function of a single school-level predictor W_k plus a random school-level residual $v_{..k}$. The random vector v_k followed a multivariate normal distribution with mean 0 and variance-covariance matrix T_β.

The approach for latent variables in this multilevel design was to represent measurement errors as a level within the hierarchical model. In essence, the level-1 model was a measurement model, whereas the next two models represented a multivariate hierarchical linear model for the latent variables. The approach could handle missing data, multiple levels of nesting, and random

efficient. "Adapting latent variable analysis for multilevel designs currently constitutes a vigorous line of methodological research" (Raudenbush & Sampson, 1999, P.124).

3.2.4 Multilevel model for repeated measures data

Less obvious applications of multilevel models are longitudinal research and growth research where several distinct observations are nested within individuals (Hox, 1995). When the same individuals are measured on more than one occasion, as usually occurs in the longitudinal studies, another hierarchically structured data occurs. The measurement repetitions are regarded as level 1 units and subjects as level 2 units. Such data are often referred to as longitudinal data, and all the repeated measurements are treated as responses.

Few studies have focused on the nature of learning as a process of change over time. Although some researchers have considered longitudinal data, they used at most two time points, with the first measure functioning as a control for prior achievement in models predicting subsequent achievement (Wilkins & Ma, 2002).

Not only students' status but also their rate of growth in one subject could be estimated by the multilevel model. Furthermore, the effects of student characteristics and school composition on students' status and rate of growth could also be examined via the multilevel model for repeated measures data.

A simple level-1 model would be

$$Y_{ij} = \alpha_{0j} + \alpha_{1j}\text{Time}_{ij} + e_{ij}. \tag{3.16}$$

This model assumes that response variables are linearly related to time within

each subject. However, growth may not be linear for all students over this age range. Non-linearity parameters such as the quadratic term need to be added to the model (Rasbash, 2002). Although adding parameters to a growth model can improve model accuracy, doing so increases the complexity of the model and should be done only when the advantages conferred by improved accuracy overweigh the disadvantages associated with greater complexity (Boyle & Willms, 2001).

A level-2 model is

$$\begin{aligned} \alpha_{0j} &= \beta_{00} + u_{oj} \\ \alpha_{1j} &= \beta_{10} + u_{1j}, \end{aligned} \tag{3.17}$$

where u_{oj} captures the difference between each person's estimated initial status α_{0j} and the average initial status β_{00}, and the residual u_{1j} captures the difference between each person's estimated rate of growth α_{1j} and the average rate of growth β_{10}.

Ma (1999) applied a three-level hierarchical linear model to examine differences in the rate at which males and females acquired mathematical skills during secondary grades. The statistical procedure is as follows:

The first level was to model students' outcome scores on their grade levels:

$$Y_{ijk} = \alpha_{0jk} + \alpha_{1jk}\text{Grade}_{ijk} + e_{ijk}, \tag{3.18}$$

where Y_{ijk}, α_{0jk}, and α_{1jk} represented the score, the initial status, and the rate of growth for jth student in kth school, respectively.

Student-level variables were added to the second-level multilevel model to model the initial status and the rate of growth.

$$\begin{aligned} \alpha_{0jk} &= \beta_{00k} + \beta_{01k}X_{jk} + u_{ojk} \\ \alpha_{1jk} &= \beta_{10k} + \beta_{11k}X_{jk} + u_{1jk}, \end{aligned} \tag{3.19}$$

where the parameters β_{00k} and β_{10k} represented the expected initial status and rates of growth for kth school after controlling student-level variables.

The third level of the model included two between-school equations that regressed the average initial status and the average rate of growth in mathematics on several school-level covariates:

$$\begin{aligned} \beta_{00k} &= \gamma_{000} + \gamma_{001} Z_k + v_{00k} \\ \beta_{10k} &= \gamma_{100} + \gamma_{101} Z_k + v_{10k}. \end{aligned} \tag{3.20}$$

The statistical analysis were completed in two steps. First, the level-one model was just a simple linear growth model without any student-level variables and school-level variables. In this simple model, the initial status and the rate of growth in mathematics were described as an average value (fixed effect) plus a variation (random effect). This approach provided an opportunity to examine not only the average values of initial status and rate of growth in mathematics, but also their variances and covariance. The estimates have been adjusted for measurement and sampling errors. The second step of analysis introduced between-student and between-school covariates, establishing a complex growth model. The purpose was to use those covariates to explain the variation between students in schools and between schools regarding the initial status and rate of growth in mathematics.

3.3 Multilevel model for correlated changes

3.3.1 Multivariate multilevel model for repeated measures data

Given that the learning of mathematics is highly related to the learning of science and the educational data are hierarchical with students nested within schools, a multivariate multilevel model is required to examine the consistency of growth across mathematics and science achievement.

A four-level hierarchical linear model (HLM) was developed by Ma (2003 [b]). The first level was a mathematical manipulation to set up a multivariate environment for the analysis, with two dichotomous variables denoting the two school subjects.

A simple measurement model for the individual might be

$$Y_{ijkl} = \alpha_{1jkl} X_{1ijkl} + \alpha_{2jkl} X_{2ijkl}, \tag{3.21}$$

where Y_{ijkl} was the total score in school subject i at grade level j for student k in school l. Without the error component, the first-level model was not a statistical model but a mathematical device to create a multivariate framework (Goldstein, 1995; Ma, 2001). The measurement model with dichotomous variables serves as a device to combine the outcomes of the two subject areas into one equation. The MLwin program allows the removal of the intercept and the errors at the first level.

The second level included two sets of separate linear regression models. The regression equations modelled scores on their grade levels in each school

subject.

$$\begin{aligned} \alpha_{1jkl} &= \beta_{10kl} + \beta_{11kl}\text{Time}_{jkl} + u_{1jkl} \\ \alpha_{2jkl} &= \beta_{20kl} + \beta_{21kl}\text{Time}_{jkl} + u_{2jkl}, \end{aligned} \tag{3.22}$$

where α_{1jkl} and α_{2jkl} were mathematics achievement scores and sciences achievement scores in grade level j for student k in school l, $Time_{jkl}$ was the grade level j of student k in school l, and u_{1jkl} and u_{2jkl} were error terms indicating the variation of each observation from the hypothesized trajectory of grade j. The parameters β_{10kl} and β_{11kl} represented the initial status and the rate of growth in mathematics achievement for student k in school l, where β_{20kl} and β_{21kl} were the corresponding parameters for growth in science achievement. The second-level models were therefore the growth models (within-student models).

The third level contained two sets of between-student regression equations modelling the rates of growth in mathematics achievement and science achievement, respectively, with student characteristics:

$$\begin{aligned} \beta_{11kl} &= \gamma_{110l} + \sum_p \gamma_{11pl} X_{pkl} + v_{11kl} \\ \beta_{21kl} &= \gamma_{210l} + \sum_q \gamma_{21pl} X_{qkl} + v_{21kl}. \end{aligned} \tag{3.23}$$

In these equations, student rate of growth was represented as a school average rate of growth (γ_{110l} and γ_{210l}), an error term (v_{11kl} and v_{21kl}), and contributions of student-level variables. Because student- and school-level variables were centered in the present study, the parameters γ_{110l} and γ_{210l} represented the rates of growth in mathematics achievement and science achievement. Other parameters represented the relationships of the rates of growth in mathematics achievement and science achievement to student-level variables (contributions of student-level variables). Obviously, the third-level models were

the student models (between-student models).

The fourth level included two between-school equations that regressed the average rates of growth in mathematics achievement and science achievement on school characteristics:

$$\begin{aligned} \gamma_{110l} &= \eta_{1100} + \sum_r \eta_{110r} W_{rl} + f_{110l} \\ \gamma_{210l} &= \eta_{2100} + \sum_s \eta_{210s} W_{sl} + f_{210l}. \end{aligned} \tag{3.24}$$

In these equations, each average rate of growth was represented as an average value (η_{1100} and η_{2100}), an error term (f_{110l} and f_{210l}), and contributions of school-level variables. Obviously, the fourth-level models were the school models (between-school models). This four-level multivariate multilevel model was tested and applied to the LSAY data on the PC platform of the MLwin program (Rasbash et al., 2000).

3.3.2 Multivariate multilevel model for repeated measures data with latent variables

Mathematics achievement was quantified as a latent variable score obtained by IRT techniques using multiple test forms and a large number of items including basic mathematics (BAS), Algebra (ALG), Geometry (GEO), and Quantitative literacy (QLT) in the LASY (Muthen, 1996). Meanwhile, Biology (BIO), Physics (PHY) and Environment (ENV) were measures of the "true" score of science achievement. We introduced a four-level hierarchical linear model to model the consistency of the growth between "true" mathematics achievement and "true" science achievement. This model could take measurement errors into account.

Specifically, the first level was a measurement model which includes latent variables. The seven items measured the latent construct for each student in each school during each year. A simple measurement model for the individual might be

$$\begin{pmatrix} Y_{1jkl} \\ Y_{2jkl} \\ Y_{3jkl} \\ Y_{4jkl} \\ Y_{5jkl} \\ Y_{6jkl} \\ Y_{7jkl} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \alpha_{1jkl} \\ \alpha_{2jkl} \end{pmatrix} + \begin{pmatrix} e_{1jkl} \\ e_{2jkl} \\ e_{3jkl} \\ e_{4jkl} \\ e_{5jkl} \\ e_{6jkl} \\ e_{7jkl} \end{pmatrix}. \tag{3.25}$$

The first four item scores, Y_{1jkl}, Y_{2jkl}, Y_{3jkl} and Y_{4jkl}, which are basic mathematics, Algebra, Geometry, and Quantitative literacy, were fallible measures of the latent score α_{1jkl} for "true" mathematical ability with measurement errors e_{1jkl}, e_{2jkl}, e_{3jkl}, and e_{4jkl} respectively. The last three item scores measured the second latent true score α_{2jkl} with errors. The measurement errors $e_{.jkl}$ were assumed independent and constant variance, σ^2. This model was a within-student model. The formulation allowed the use of all available data in the analysis.

The second level of the model was a set of separate linear regression models, one for each student. The regression equations modelled students' latent outcome scores at their grade levels, which described the variation in the two latent variable within grades. The grade-level models were as follows:

$$\begin{aligned} \alpha_{1jkl} &= \beta_{10kl} + \beta_{11kl}\text{Grade}_{jkl} + u_{1jkl} \\ \alpha_{2jkl} &= \beta_{20kl} + \beta_{21kl}\text{Grade}_{jkl} + u_{2jkl}, \end{aligned} \tag{3.26}$$

where $Grade_{jkl}$ was the grade that student k in school l at the testing time l, and u_{1jkl} and u_{2jkl} were error terms. The parameter β_{10kl} and β_{11kl} represented the initial status and the rate of growth in latent mathematics achievement for student k in school l, whereas β_{20kl} and β_{21kl} were the corresponding parameters for growth in latent science achievement. The second-level models were therefore the growth models (within-student models).

Similar to the previous multivariate multilevel model for repeated measures data, the third level model contained two sets of between-student regression equations modelling the rates of growth in mathematics and science achievement, respectively, with student characteristics:

$$\begin{aligned} \beta_{11kl} &= \gamma_{110l} + \sum_p \gamma_{11pl} X_{pkl} + v_{11kl} \\ \beta_{21kl} &= \gamma_{210l} + \sum_q \gamma_{21pl} X_{qkl} + v_{21kl}. \end{aligned} \tag{3.27}$$

School-level models were as follows:

$$\begin{aligned} \gamma_{110l} &= \eta_{1100} + \sum_r \eta_{110r} W_{rl} + f_{110l} \\ \gamma_{210l} &= \eta_{2100} + \sum_s \eta_{210s} W_{sl} + f_{210l}. \end{aligned} \tag{3.28}$$

The details will be provided in the next chapter.

3.4 Parameter estimation

Iterative Generalized Least Squares (IGLS) or Restricted Iterative Generalized Least Squares (RIGLS) are the basis of parameter estimation methods for multilevel models. The following is an overview of how IGLS works in a linear model (Goldstein, 1986; Goldstein, 1995).

Consider the simple 2-level model:

$$Y_{ij} = \alpha_{0j} + \alpha_1 x_{ij} + e_{ij}, \tag{3.29}$$

where

$$\alpha_{0j} = \beta_{00} + u_{0j}, \tag{3.30}$$

The combined model is as follows:

$$Y_{ij} = \beta_{00} + \alpha_1 x_{ij} + u_{0j} + e_{ij}. \tag{3.31}$$

The model could be rewritten as

$$Y = X\beta + U + E, \tag{3.32}$$

where in this case $Y = \begin{pmatrix} Y_{11} \\ Y_{21} \\ \vdots \\ Y_{m_n m} \end{pmatrix}$ and $X = \begin{pmatrix} 1 & x_{11} \\ 1 & x_{21} \\ \vdots & \vdots \\ 1 & x_{m_n m \cdot} \end{pmatrix}$

Suppose that the variance of Y is known, namely V, the estimator for the fixed coefficients could be obtained by the usual Generalized Least Square (GLS) estimation procedure.

$$\hat{\beta} = (X^T V^{-1} X)^{-1} X^T V^{-1} Y \tag{3.33}$$

If β is known but V is unknown, then we can obtain estimators $\hat{\beta}^*$ of the parameters of V using GLS as

$$\hat{\beta}^* = (X^{*T}(V^*)^{-1}X^*)^{-1} X^{*T}(V^*)^{-1} Y^*, \tag{3.34}$$

where Y^* is the vector of the upper triangle element of $(Y - X\beta)(Y - X\beta)^T$, which is the squares and products of the residuals; V^* is the covariance matrix of Y^*; and X^* is the design matrix linking Y^* to V in the regression of Y^* on X^*.

However, neither V nor β is known, so the iterative generalized least square estimates are those which simultaneously satisfy both equation (3.33) and

(3.34). The IGLS estimation is iterative. Usually, it starts with reasonable estimates of the fixed coefficients from an initial Ordinary Least Square (OLS) fit. The random parts U and E are treated together as a residual, therefore V is assumed to be σ^2. The OLS estimates of the fixed coefficient β_0 is obtained by

$$\widehat{\beta_0} = (X^T X)^{-1} X^T Y. \tag{3.35}$$

Based on the initial estimate of V, an estimate of β could be obtained, and then an improved estimate of β would be obtained, and so on, iteratively, until convergence is achieved. Essentially the same procedure can be used for the more complicated models and is incorporated in the program MLwin.

RIGLS allows us to choose a convergence criterion value, for example 10^{-3}. Then the relative change from one iteration to the next has to be less than that for all parameters in order to achieve the convergence.

Estimation for the hierarchical model could also be implemented via a Fisher scoring method or the EM algorithm (Raudenbush, 1988; Raudenbush et al., 1991).

Chapter 4

Data Analysis

The present study examined the growth in latent mathematics achievement and growth in latent science achievement, attempting to discover whether students who grew quickly in "true" mathematics achievement also did so in "true" science achievement and what student and school characteristics influenced this consistency (or inconsistency). We also examined this issue from the schools' perspective. Specifically, two principal research questions were investigated:

1. Do students (schools) have consistent rates of growth across mathematics and science achievement? That is, if students (schools) improve quickly in one area, do they also improve quickly in the other area?

2. After adjusting for student and school characteristics, do students (schools) have consistent rates of growth across mathematics and science achievement? That is, how do student- and school- characteristics affect the correlation in rates of growth between mathematics and science achievement?

4.1 Preliminary analysis

Descriptive statistics for the four subject scores – Basic Mathematics, Algebra, Geometry and Quantitative literacy, and the other three subject scores – Biology, Physics and Environmental Sciences, are reported in Table 4.1. Item scores in each subject were imputed scores which included non-aberrant observed scores when available. These items were stored as continuous variables in the data file. The first four subjects were regarded as a measurement of mathematics achievement. The other three measured science achievement. Across grade levels, the growth of students in the four subjects relating to mathematics achievement ranged from 0.98 points to 6.36 points, while the growth of students in the other three subjects measuring science achievement ranged from 0.90 points to 11.54 points annually. Most standard deviations for mathematics achievement and science achievement also increased from Grade 7 to Grade 12. This finding indicates that the achievement gap among students increased across grade levels in both mathematics achievement and science achievement.

Table 4.1 Mean and standard deviation of outcome measures

Mathematics achievement

	BAS		ALG		GEO		QLT	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Grade 7	50.41	10.24	50.23	10.08	50.29	10.27	50.31	10.12
Grade 8	54.80	11.52	56.32	16.95	55.91	13.03	54.76	11.69
Grade 9	60.13	13.26	67.86	19.40	64.06	15.83	60.16	12.85
Grade 10	64.31	13.65	76.30	20.95	72.42	19.20	64.83	13.95
Grade 11	68.51	13.65	85.10	22.21	79.39	20.11	68.80	13.90
Grade 12	69.64	14.28	89.47	25.66	80.44	22.05	69.71	16.30

Science achievement

	BIO		PHY		ENV	
	Mean	SD	Mean	SD	Mean	SD
Grade 7	50.35	10.08	50.39	10.22	50.44	10.28
Grade 8	53.12	12.30	54.98	11.38	53.13	13.38
Grade 9	59.48	11.81	58.62	11.63	59.35	13.51
Grade 10	62.93	12.27	61.71	12.43	63.69	14.07
Grade 11	66.07	12.41	64.94	12.93	66.65	14.11
Grade 12	67.68	14.71	67.56	13.70	67.63	15.51

The descriptive statistics for student-level variables, which are gender, age, mother??s SES, father??s SES, number of siblings, number of parents, and race-ethnicity, are listed in the Table 4.2.

Table 4.2 Descriptive statistics for student-level variables

	Number	Minimum	Maximum	Mean	Std. Dev.
GENDER	3116	.00	1.00	.48	.50
AGE (month)	3056	103.00	195.00	152.83	7.83
MOTHSEI	2341	12.00	88.00	41.68	16.96
FATHSEI	2289	12.00	89.00	41.53	20.62
SIBLINGS	2481	1.00	9.00	2.88	1.40
NUMPAR	2481	.00	1.00	.78	.42
HISPANIC	2958	.00	1.00	.10	.29
BLACK	2958	.00	1.00	.12	.32
ASIAN	2958	.00	1.00	.04	.19
OTHER	2958	.00	1.00	.02	.13

Note: Gender, the number of parents, and race-ethnicity, which are Hispanic, Black, Asian, and Others, are dichotomous variables.

The descriptive statistics for school-level variables including both school context characteristics and school climate variables are reported in Tables 4.3.

Table 4.3 Descriptive statistics for student-level variables

	Number	Minimum	Maximum	Mean	Std. Dev.
EXPERIEN	52	2.50	27.00	12.46	5.68
EDLEVEL	52	16.00	18.00	16.99	.76
PRILED	52	2.21	5.00	3.98	.66
ACKPRS	52	2.20	5.11	4.08	.61
DISCLM	52	1.62	3.67	2.90	.42
TCHATU	52	2.42	4.89	3.58	.62
TCHCOM	52	3.00	6.00	4.42	.67
STFCOP	52	2.33	5.00	4.24	.50
GERASUPT	52	1.60	3.80	2.83	.38
SUBURBAN	52	.00	1.00	.42	.50
RURAL	52	.00	1.00	.33	.47
HOMEWORK	52	.95	5.00	2.73	.90
Zscore(VWS)	52	-1.53	3.30	.00	1.00
SCHSIZE	52	66.00	1450.00	612.06	283.79
FREELUN	52	.00	89.00	29.25	21.53
GRASPAN	52	1.00	12.00	2.71	2.00
STRATIO	52	2.61	13.89	6.58	2.41
PROMINO	52	.00	96.54	22.39	24.23
COMPMATH	52	.00	.11	.01	.02
EXTRACUM	52	.00	5.00	1.43	1.32

Note. Two variables describing the school location, suburban and rural, are dichotomous variables. VWS is a standardized variable at the school level.

Background variables were either standardized to have a mean of 0 and a standard deviation of 1 (for mother's and father's SES) or centered on the grand mean (for all other variables). To obtain estimates for what is often referred to as the "typical" student with nationally average characteristics, all background variables need to be centered (Bryk & Raudenbush, 1992).

4.2 Multilevel models from MLwin

MLwiN is a windows-based statistical software package developed by the Multilevel Models Project for the analysis of multilevel models. It can fit models to hierarchical data from the two-level random-effects model with a Normal Continuous Response to more complex multi-level extensions. We applied the multivariate multilevel model to the repeated measures data with latent variables. We applied the multivariate multilevel model for repeated measures data with latent variables on the LSAY data with the use of MLwin.

4.2.1 Null model

We performed the analysis in three steps. In the first step, a simple model was estimated without student-level variables and school-level variables. In such a simple model, the rates of growth in mathematics achievement and science achievement were described as an average value (fixed effect) plus a variation (random effect) at both the student level and the school level. The equations (3.25) and (3.26) remained the same, however. In this case, the student-level

equations were written as follows:

$$\begin{aligned} \beta_{11kl} &= \gamma_{110l} + v_{11kl} \\ \beta_{21kl} &= \gamma_{210l} + v_{21kl}, \end{aligned} \tag{4.1}$$

where the parameters β_{11kl} and β_{21kl} represented the separate growth trajectory of mathematics achievement and science achievement for each student in each school. γ_{110l} and γ_{210l} represented the average growth trajectory of mathematics achievement and science achievement for each school. The growth trajectory for each student deviated from this average growth trajectory; as a result, the residuals v_{11kl} and v_{21kl} captured the differences between each student's rate of growth and the average rate of growth in mathematics achievement and science achievement, respectively.

Although the estimate of the slope was of primary interest, β_{10kl} and β_{20kl} were important, too. We had not only random slopes but also random intercepts, so that

$$\begin{aligned} \beta_{10kl} &= \gamma_{100l} + v_{10kl} \\ \beta_{20kl} &= \gamma_{200l} + v_{20kl}. \end{aligned} \tag{4.2}$$

The random vector $\begin{pmatrix} v_{10kl} \\ v_{11kl} \\ v_{20kl} \\ v_{21kl} \end{pmatrix}$ followed a normal distribution with mean vector 0 and variance covariance Ω_v.

The fundamental idea underlying multilevel modelling is that the parameters from one level become the response variables at the next level. In this

way, the school-level model could be written as follows:

$$\begin{aligned} \gamma_{110l} &= \eta_{1100} + f_{110l} \\ \gamma_{210l} &= \eta_{2100} + f_{210l}, \end{aligned} \tag{4.3}$$

where the parameter η_{1100} and η_{2100} represented the overall average rates of growth of mathematics achievement and science achievement for all students in all schools. The residual f_{110l} and f_{210l} captured the differences between each school's estimated growth trajectory and the average growth trajectory in mathematics achievement and science achievement.

Also for the same reason, we could write the school-level equations for the initial status from the student level:

$$\begin{aligned} \gamma_{100l} &= \eta_{1000} + f_{100l} \\ \gamma_{200l} &= \eta_{2000} + f_{200l}. \end{aligned} \tag{4.4}$$

The random vector $\begin{pmatrix} f_{100l} \\ f_{110l} \\ f_{200l} \\ f_{210l} \end{pmatrix}$ followed a normal distribution with mean vector 0 and variance covariance Ω_f.

The variance and covariance at the third and fourth levels associated with the rates of growth in mathematics achievement and science achievement were used to calculate the correlation coefficient between the rates of growth in mathematics achievement and science achievement among students and among schools. The correlation coefficient was used as a measure of the stability of growth between mathematics achievement and science achievement at the student and school levels. Following Goldstein (1987), the correlation coefficient

was calculated as

$$\rho = \frac{\sigma_{i,i'}}{\sqrt{\sigma_i^2 \sigma_{i'}^2}} = \frac{\sigma_{i,i'}}{\sigma_i \sigma_{i'}},$$

where σ_i^2 and $\sigma_{i'}^2$ were the variance (of either students and schools) on subject area i and i', and $\sigma_{i,i'}$ was the covariance between the two subject areas i and i'. The variance and covariance can be adjusted for student and school characteristics.

Table 4.4 Results from MLwin for the null model

Fixed effects

	Mathematics achievement		Science achievement	
	Effect	SE	Effect	SE
Initial status	50.715	0.559	50.782	0.528
Rate of change	5.135	0.139	3.101	0.085

Variance covariance components

$$\Omega_v = \begin{pmatrix} 55.723 & 0.765 & 0.591 & 0.395 \\ 45.634 & 63.9 & 0.580 & 0.309 \\ 10.420 & 10.956 & 5.582 & 0.58 \\ 4.720 & 3.949 & 2.192 & 2.561 \end{pmatrix}$$

$$\Omega_f = \begin{pmatrix} 14.965 & 0.876 & 0.698 & 0.525 \\ 12.246 & 13.056 & 0.713 & 0.494 \\ 2.444 & 2.332 & 0.820 & 0.907 \\ 1.050 & 0.922 & 0.424 & 0.267 \end{pmatrix}$$

Note. The lower triangles of these matrices contain the variance and covariance; the upper triangles contain the correlations.

In conclusion, the unadjusted model contained neither student-level variables nor school-level variables. The results from MLwin are listed in Table 4.4 (see also Figures 1∼3.) Hence, the correlation between the rates of growth among students is reported as 0.58, whereas the correlation between the rates of growth among schools is reported as 0.91.

4.2.2 Between-student model

In the second step, we introduced student-level variables to explain the variation between students in the rates of growth in mathematics achievement and science achievement. As a multiple regression, the student variable were added to the null model separately. Later only the significant student variable were kept to examine which one had a significant effect on the growth rate in the presence of other variables.

The absolute and relative effects for student and school variables are shown in Table 4.5. The absolute effects provided a measure of the independent effects of student variables. The relative effects were adjusted for the other variables in the model. Gender, age, number of parents and Asian had both significant absolute and relative effects on the rate of growth in mathematics achievement.

The equations (3.25) and (3.26)remained the same. However, the third-level models were written as follows:

$$\begin{aligned}\beta_{10kl} &= \gamma_{100l} + 0.174\text{Gender} - 0.058\text{Age} + 0.279\text{Numpar} + 0.734\text{Asian} + \upsilon_{10kl} \\ \beta_{20kl} &= \gamma_{200l} - 0.583\text{Black} + \upsilon_{20kl}.\end{aligned} \tag{4.5}$$

The fourth-level model turned out to be

$$\begin{aligned} \gamma_{110l} &= \eta_{1100} + f_{110l} \\ \gamma_{210l} &= \eta_{2100} + f_{210l}, \end{aligned} \tag{4.6}$$

Thus, within students, the rate of growth in mathematics was viewed as dependent on students' gender, age, number of parents and their racial ethnicity. Meanwhile, the rate of growth in science was viewed as dependent on students' racial ethnicity. Of center interest in this analysis were random effects v_{10kl} and v_{20kl} at the student level, as well as the random effects f_{110l} and f_{210l} at the school level.

Table 4.5 Student variables' effect on the grow trajectory

	Mathematics achievement		Science achievement	
	Absolute	Relative	Absolute	Relative
Gender	0.247 (0.089)	0.174 (0.089)	-0.155 (0.072)	-
Age	-0.058 (0.007)	-0.058 (0.007)	-0.008 (0.006)	-
Mother's SES	0.084 (0.045)	-	0.041 (0.036)	-
Father's SES	0.022 (0.046)	-	0.037 (0.037)	-
No. of Siblings	0.014 (0.035)	-	-0.028 (0.029)	-
No. of parents	0.251 (0.119)	0.279 (0.118)	0.126 (0.097)	-
Hispanic	-0.250 (0.160)	-	0.023 (0.129)	-
Black	-0.124 (0.147)	-	-0.528 (0.118)	-0.583 (0.119)
Asian	0.586 (0.219)	0.734 (0.217)	-0.097 (0.176)	-
Other	-0.541 (0.340)	-	0.167 (0.275)	-

After adjusting for student-level variables, according to the results from the MLwin, the results are listed in the following Table 4.6 (see also Figure 4~6).

Table 4.6 Results from MLwin for between-student model

Fixed effects

	Mathematics achievement		Science achievement	
	Effect	SE	Effect	SE
Initial status	50.734	0.558	50.777	0.528
Rate of change	5.101	0.132	3.109	0.080

Variance covariance components

$$\Omega_v = \begin{pmatrix} 55.557 & 0.764 & 0.567 & 0.382 \\ 45.541 & 63.915 & 0.567 & 0.296 \\ 9.588 & 10.294 & 5.152 & 0.560 \\ 4.519 & 3.752 & 2.014 & 2.513 \end{pmatrix}$$

$$\Omega_f = \begin{pmatrix} 14.915 & 0.876 & 0.625 & 0.435 \\ 12.232 & 13.062 & 0.689 & 0.375 \\ 2.071 & 2.135 & 0.736 & 0.851 \\ 0.793 & 0.641 & 0.345 & 0.223 \end{pmatrix}$$

Note. The lower triangles of these matrices contain the variance and covariance; the upper triangles contain the correlations.

In this model, the correlation between the rates of growth among students is reported as 0.56, whereas the correlation between the rates of growth among schools is 0.85.

4.2.3 Between-school model

In the third step, both student-level and school-level variables were added to explain the variation between students and between schools in the rates of

growth in mathematics achievement and science achievement (see Table 4.7).

Table 4.7 School variables' effect on the grow trajectory

	Mathematics achievement		Science achievement	
	Absolute	Relative	Absolute	Relative
EXPERIEN	-0.011 (0.011)	-	0.009 (0.008)	-
EDLEVEL	0.126 (0.087)	-	-0.024 (0.064)	-
PRILED	0.102 (0.102)	-	-0.067 (0.074)	-
ACKPRS	0.250 (0.112)	-	-0.118 (0.074)	-
DISCLM	0.216 (0.170)	-	0.051 (0.126)	-
TCHATU	-0.018 (0.108)	-	0.093 (0.077)	-
TCHCOM	0.240 (0.096)	0.269 (0.088)	-0.086 (0.072)	-
STFCOP	0.163 (0.146)	-	-0.037 (0.109)	-
GERASUPT	0.225 (0.172)	-	0.187 (0.123)	-
SUBURBAN	0.306 (0.127)	-	-0.102 (0.095)	-
RURAL	-0.300 (0.134)	-	0.229 (0.096)	0.192 (0.088)
HOMEWORK	-0.050 (0.077)	-	0.007 (0.057)	-
Zscore(VWS)	0.018 (0.072)	-	0.020 (0.053)	-
SCHSIZE	0.000 (0.000)	-	0.000 (0.000)	-
FREELUN	-0.001 (0.003)	-	-0.005 (0.002)	-0.006 (0.002)
GRASPAN	0.002 (0.033)	-	0.004 (0.024)	-
STRATIO	-0.047 (0.028)	-	0.037 (0.020)	-
PROMINO	0.002 (0.003)	-	-0.002 (0.002)	-
COMPMATH	-7.643 (3.303)	-7.663 (3.129)	5.875 (2.403)	-
EXTRACUM	0.000 (0.048)	-	0.017 (0.035)	-

The equations (3.25) and (3.26) remained the same for the null model. The third-level models turned to be (4.5) and the fourth-level model was written

as follows:

$$\gamma_{110l} = \eta_{1100} + 0.269\text{TCHCOM} - 7.663\text{COMPMATH} + f_{110l}$$
$$\gamma_{210l} = \eta_{2100} + 0.192\text{RURAL} - 0.006\text{FREELUN} + f_{210l}, \tag{4.7}$$

After adjusting for both student-level variables and school-level variables, the results from the MLwin are as listed in Table 4.8 (see also Figure 7~9).

Table 4.8 Results from MLwin for between-school model

Fixed effects

	Mathematics achievement		Science achievement	
	Effect	SE	Effect	SE
Initial status	50.726	0.557	50.777	0.527
Rate of change	5.101	0.128	3.109	0.073

Variance covariance components

$$\Omega_u = \begin{pmatrix} 55.574 & 0.764 & 0.567 & 0.384 \\ 45.537 & 63.888 & 0.568 & 0.297 \\ 9.602 & 10.307 & 5.160 & 0.559 \\ 4.533 & 3.762 & 2.013 & 2.509 \end{pmatrix}$$

$$\Omega_f = \begin{pmatrix} 14.915 & 0.876 & 0.681 & 0.320 \\ 12.191 & 13.029 & 0.705 & 0.195 \\ 2.156 & 2.092 & 0.676 & 0.813 \\ 0.513 & 0.292 & 0.278 & 0.173 \end{pmatrix}$$

Note. The lower triangles of these matrices contain the variance and covariance; the upper triangles contain the correlations.

In this model, the correlation between the rates of growth among students is reported as 0.56, whereas the correlation between the rates of growth among

schools is 0.81.

Three models were used to estimate the correlation coefficients between the rates of growth in mathematics achievement and science achievement, and the third model was adjusted for both student-level variables and school-level variables.

Chapter 5

Results

From the null model, students were found to have grown on average 5.135 and 3.101 points annually in the "true" mathematics achievement and the "true" science achievement, respectively.

After controlling for student characteristics, "typical" students were found to have grown 5.101 and 3.109 points annually in the "true" mathematics achievement and the "true" science achievement, respectively. Female students were found to have improved more quickly than male students in the "true" mathematics achievement, and younger students were found to have grown faster than older students. It was also found that students from both-parents families grew faster than student from single-parent families in the "true" mathematics achievement, and Asian students grew faster than White students in the "true" mathematics achievement. White students were found to have improved at a faster rate than Black student in the "true" science achievement.

After controlling for student characteristics and school characteristics, "typical" students were found to have grown 5.101 points annually in the "true"

mathematics achievement, which remained the same. At the school level, significant effects were associated with teacher commitment and computer-student ratio for the rate of growth in the "true" mathematics achievement. Teacher commitment had a positive effect on the growth trajectory of the "true" mathematics achievement, while the computer student ratio had a negative effect on the growth trajectory of the "true" mathematics achievement.

Moreover, after controlling for student characteristics and school characteristics, students were found to have grown 3.109 points annually in the "true" science achievement, which was also the same as the between-student model. At the school level, students in schools with a lower percentage of free-lunch students grew in the "true" science achievement at a faster rate than students in schools with a higher percentage of free-lunch students. Students from rural area had a faster growth rate in the "true" science achievement.

However, the correlation between the estimate of slopes was of primary interest. Table 5.1 presents the correlation coefficients between the rates of growth in the "true" mathematics achievement and science achievement from the three models. The correlation coefficient between the rates of growth in the "true" mathematics achievement and the "true" science achievement from the null model was 0.58 at the student level. A correlation coefficient of this magnitude can be described as rather weak, indicating that students who grew fast in the "true" mathematics achievement did not necessarily grow fast in the "true" science achievement. After the rates of growth in the "true" mathematics achievement and the "true" science achievement were adjusted for student-level and school-level variables, the correlation coefficient became 0.56, representing a trivial change from 0.58 from the unadjusted model. The

addition of school-level variables to further adjust the rates of growth in the "true" mathematics achievement and the "true" science achievement did not influence the relationship between the rates of growth in the "true" mathematics achievement and the "true" science achievement at all, with the correlation coefficient remaining at 0.56 (the same as that from the model adjusted for student-level variables alone). Therefore, we concluded that the correlation coefficient between the rates of growth in the "true" mathematics achievement and the "true" science achievement was rather weak among students, and that this inconsistency was not influenced much by student-level variables and school-level variables.

The correlation coefficient between the average rates of growth in the "true" mathematics achievement and the "true" science achievement from the unadjusted model was 0.91 among schools. This correlation coefficient can be considered strong. This result indicates that schools that grew fast in the "true" mathematics achievement also grew fast in the "true" science achievement. After the average rates of growth in the "true" mathematics achievement and the "true" science achievement were adjusted for student-level variables, the correlation coefficient dropped to 0.85, indicating a moderate departure from 0.91, the correlation efficient from the null model. The addition of school-level variables to further adjust the average rates of growth in the "true" mathematics achievement and the "true" science achievement further reduced the correlation coefficient to 0.81, indicating a sizable departure, especially from 0.91, the correlation coefficient from the null model. Therefore, we concluded that the correlation coefficient between the average rates of growth in the "true" mathematics and the "true" science achievement was strong among

schools, and that this consistency was influenced by student-level variables and school-level variables.

Moreover, the relationship between the rates of growth in the "true" mathematics achievement and the "true" science achievement was much stronger at the school level than at the student level across all three models. For the unadjusted model, the correlation among schools was much stronger than the one among students. Although the addition of student-level variables and school-level variables reduced the correlation coefficient among schools, it remained much stronger than that among students. We concluded that the relationship between the rates of growth in the "true" mathematics achievement and the "true" science achievement was much more evident among schools than among students (see Table 5.1).

Table 5.1 Correlation between the rates of growth in the "true" mathematics and the "true" science achievement

	Among students		Among schools	
	M	S	M	S
Unadjusted				
Mathematics (M)	1.00		1.00	
Science (S)	0.58	1.00	0.91	1.00
Adjusted for student-characteristics				
Mathematics (M)	1.00		1.00	
Science (S)	0.56	1.00	0.85	1.00
Adjusted for student- and school-characteristics				
Mathematics (M)	1.00		1.00	
Science (S)	0.56	1.00	0.81	1.00

Chapter 6

Conclusions and Discussion

The consistency in the rates of growth in the "true" mathematics achievement and the "true" science achievement among students and schools was examined by the multivariate multilevel model in the present study, with student-level variables and school-level variables available for adjustment. The within-student model at the first-level was used as an approach to link the items with the latent variables measuring the "true" mathematics achievement and the "true" science achievement. The first four item scores, which were Basic Mathematics, Algebra, Geometry, and Quantitative literacy, were fallible measures of the latent score for "true" mathematical ability with measurement errors. The other three item scores, which were Biology, Physics, and Environment, measured the "true" science ability with measurement errors. The growth model at the second-level was adopted to examine the rates of the growth in the "true" mathematics achievement and the "true" science achievement, which was our interest. Each of the regression coefficients from the second-level were modelled as a dependent variables through student variables (gen-

der, age, race-ethnicity, mother's SES, father's SES, number of parents, and number of siblings). Since the basic idea underlying multilevel modelling is that the parameters from one level become the outcome variables at the next level, the regression constants (intercepts) at the third level was then modelled through the school variables (EXPERIEN, EDLEVEL, PRILED, ACKPRS, DISCLM, TCHATU, TCHCOM, STFCOP, GERASUPT, SUBURBAN, RURAL, HOMEWORK, VWS, SCHSIZE, FREELUN, GRASPAN, STRATIO, PROMINO, COMPMATH, and EXTRACUM).

We did not find evident consistency in the rates of growth between "true" mathematics achievement and "true" science achievement among students, and we found that this consistency was not much affected by student-level variables and school-level variables. However, we did find evident consistency in the average rates of growth of the "true" mathematics achievement and the "true" science achievement among schools, and we found that this consistency was influenced by student-level variables and school-level variables. In addition, we found that the relationship between the rates of growth in the "true" mathematics achievement and the "true" science achievement was much more evident at the school level than at the student-level.

Sammons et al. (1997) and Thomas et al. (1997) found that the correlation among students between mathematics achievement and science achievement was 0.35 at the secondary school level. These researchers considered this coefficient rather weak, indicating that students who do well in mathematics may not necessarily do well in science. In the present study, we added the developmental perspective to these findings. We found that correlation between the rates of growth in the "true" mathematics achievement and the "true" science

achievement was 0.56 among students (after adjusting for student-level variables and school-level variables). In magnitude, our coefficient was not much stronger than that of the previous researchers (0.35), indicating that students who progress well in the "true" mathematics achievement may not necessarily progress well in the "true" science achievement. We agree that students were differentially successful in learning mathematics and science.

These are puzzling findings because it is well-known that the learning of science requires strong mathematical skills. Recall that the science achievement test in the LSAY measured three components: biology, physics, and environmental sciences. We think that students do not need strong mathematical skills to do well in biology and, especially, in environmental sciences. However, we suspect that fast progress in mathematical skill can help students learn "hard" science, namely physics. We encourage further studies to look into the hypothesis that faster growth in mathematics skills promotes faster growth in achievement in certain science areas. In general, these findings raise the issue that the possibility between mathematics and science learning may depend on the nature of the curricula in mathematics and science.

We also suggest that this inconsistency in the rates of growth between mathematics achievement and science achievement is a common phenomenon among students because we found that adjustment for student-level variables did not significantly change this inconsistency. From an individual perspective, even school-level variables did not significantly change this inconsistency. These findings indicated that the fact that some students did improve faster in both the "true" mathematics achievement and the "true" science achievement had not much to do with student characteristics and school characteristics. In

other words, these students did not form groups (at the student level) or come from schools that shared reasonably common characteristics.

Although few studies exist that look into the consistency of school effects (Thomas et al., 1997), their findings largely agree with one another. For example, Luyten (1994) argued that school departments were differentially effective in their teaching, and Ma (2001) reported that both students and schools were differentially successful in learning and teaching mathematics and science. From a developmental perspective (growth in mathematics achievement and science achievement across the entire secondary school), however, the schools did appear equally effective in teaching mathematics and science.

We found a strong relationship between the average rates of growth in the "true" mathematics achievement and the "true" science achievement among schools. Schools with a faster average rate of growth in the "true" mathematics achievement also showed a faster average rate of growth in the "true" science achievement. Therefore, we argue that the school mathematics and science departments (or mathematics and science teachers) were equally effective in promoting growth in the "true" mathematics achievement and the "true" science achievement.

This consistency among schools in the average rates of growth between the "true" mathematics achievement and the "true" science achievement was influenced by student-level variables and school-level variables. Therefore, student characteristics and school characteristics contributed in an important way to this consistency. Table 4.5 and 4.7 offer insights into those important student characteristics and school characteristics.

Among student background characteristics examined in this study, Asian

was the most significant predictor of the rate of growth in the mathematical skills. Asian students were found to have a faster growth rate in Mathematics than white students. Gender was also a significant predictor for the rate of growth in the "true" mathematical achievement, where female students were found to have improved more quickly than male students. Number of parents had a effect on the growth trajectory in the "true" mathematical achievement. This study indicated that the students from single-parent family were disadvantaged in the development of mathematical skills. Older students in the same grade cohort were also disadvantaged in the development of mathematical skills. Among student-level characteristics for the rate of growth in the "true" science achievement, only Black was the significant predictor. Compared to white students, black students thus need more help in their study of science.

School background characteristics – teacher commitment and computer student ratio had a significant on the rate of growth in the "true" mathematics achievement. Teacher commitment variable had a positive effect on the growth trajectory, therefore, this finding implies that schools should seek systematic coordination and improve cooperation among department and teachers. The computer student ratio had a significant negative effect on the growth trajectory of the "true" mathematics achievement. The students in schools with more computer were disadvantaged in the development of mathematical skills. It suggests that schools should allow students use the computer appropriately and give students more traditional mathematical lectures and enough practice rather than mathematical computer software.

At the school level, rural and the percentage of free lunch had effects on the

rate of growth in the "true" science achievement. Students in schools located in rural area were found to have grown at a significantly faster rate than students in urban schools. This indicates that rural schools may provide a far better chance for students to improve quickly in science or the students in rural schools have more capacity for growth and development. Student in schools with a lower percentage of free-lunch were found to have grown at a faster rate than student in schools with a higher percentage of free-lunch students in the "true" science achievement. The students in schools with lower socioeconomic status were disadvantaged in the development of their scientific skills. Our findings suggest that eliminating student characteristics gaps in the learning of mathematics and science and counterbalancing the negative impacts of school context improve the consistency in the average rates of growth between the "true" mathematics achievement and the "true" science achievement among schools.

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$$\text{score}_{ijkl} \sim N(XB, \Omega)$$

$$\text{score}_{ijkl} = \beta_{0ijkl}\text{math}_{ijkl} + \beta_{1ijkl}\text{scie}_{ijkl} + \beta_{2kl}\text{mathtime}_{ijkl} + \beta_{3kl}\text{scietime}_{ijkl}$$

$$\beta_{0ijkl} = 50.715(0.559) + f_{0l} + v_{0kl} + u_{0jkl} + e_{0ijkl}$$

$$\beta_{1ijkl} = 50.782(0.528) + f_{1l} + v_{1kl} + u_{1jkl} + e_{1ijkl}$$

$$\beta_{2kl} = 5.135(0.139) + f_{2l} + v_{2kl}$$

$$\beta_{3kl} = 3.101(0.085) + f_{3l} + v_{3kl}$$

$$\begin{bmatrix} f_{0l} \\ f_{1l} \\ f_{2l} \\ f_{3l} \end{bmatrix} \sim N(0, \Omega_f) : \Omega_f = \begin{bmatrix} 14.965(3.189) & & & \\ 12.246(2.802) & 13.056(2.839) & & \\ 2.444(0.660) & 2.332(0.631) & 0.820(0.195) & \\ 1.050(0.375) & 0.923(0.346) & 0.424(0.108) & 0.267(0.073) \end{bmatrix}$$

$$\begin{bmatrix} v_{0kl} \\ v_{1kl} \\ v_{2kl} \\ v_{3kl} \end{bmatrix} \sim N(0, \Omega_v) : \Omega_v = \begin{bmatrix} 55.723(1.840) & & & \\ 45.634(1.631) & 63.900(2.027) & & \\ 10.420(0.482) & 10.956(0.526) & 5.582(0.228) & \\ 4.720(0.376) & 3.949(0.391) & 2.192(0.138) & 2.561(0.138) \end{bmatrix}$$

$$\begin{bmatrix} u_{0jkl} \\ u_{1jkl} \end{bmatrix} \sim N(0, \Omega_u) : \Omega_u = \begin{bmatrix} 3.589(0.425) & \\ 5.503(0.292) & 13.990(0.389) \end{bmatrix}$$

$$\begin{bmatrix} e_{0ijkl} \\ e_{1ijkl} \end{bmatrix} \sim N(0, \Omega_e) : \Omega_e = \begin{bmatrix} 81.317(0.589) & \\ 0.000(0.000) & 26.556(0.236) \end{bmatrix}$$

*-2*loglikelihood(IGLS)* = 630312.400(88663 of 88663 cases in use)

Figure 1. Coefficient estimation for the null model from MLwin

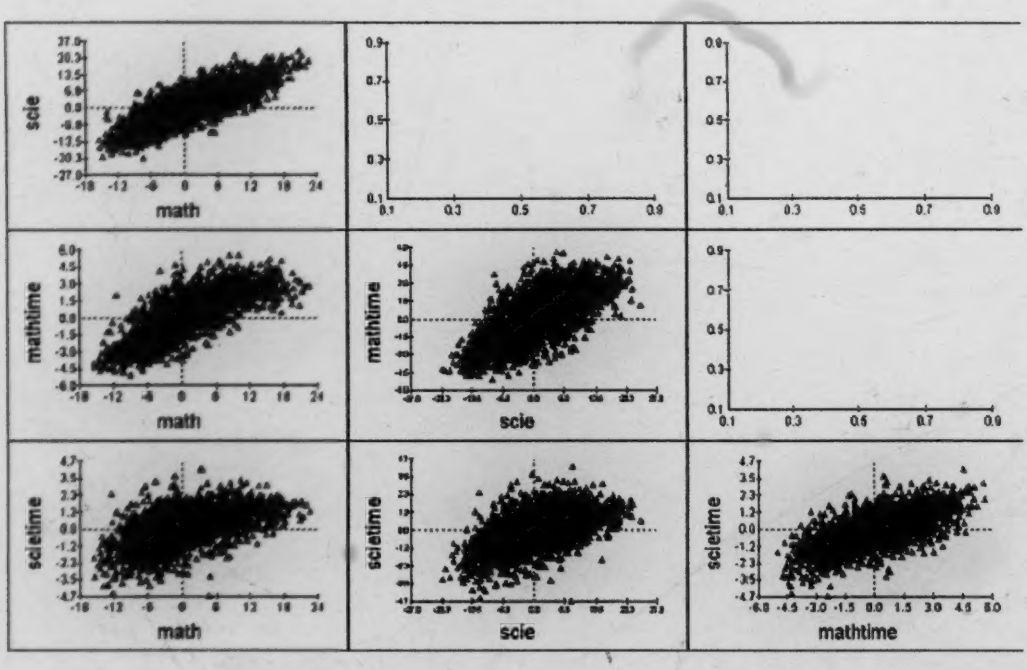

Figure 2. Scatter plot of the residuals at student level in the null model

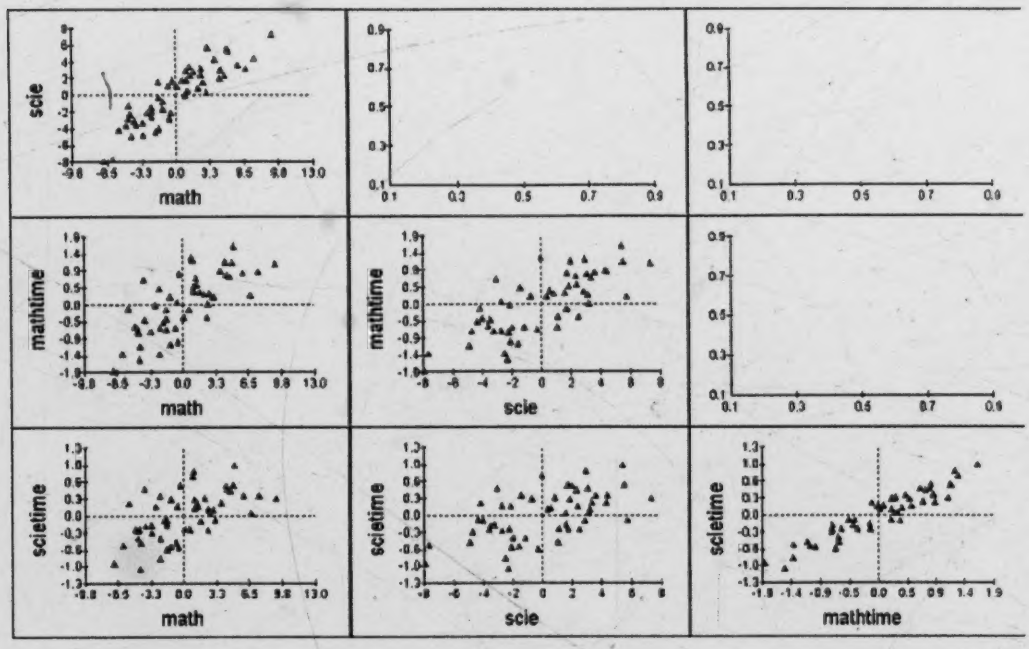

Figure 3. Scatter plot of the residuals at school level in the null model

$$score_{ijkl} \sim N(XB, \Omega)$$

$$score_{0ijkl} = \beta_{0ijkl}math_{0ijkl} + \beta_{1ijkl}scie_{ijkl} + \beta_{2kl}mathtime_{ijkl} + \beta_{3kl}scietime_{ijkl} + -0.058(0.007)mtage_{ijkl} + 0.279(0.118)mtparents_{ijkl} + 0.724(0.217)mtasian_{ijkl} + 0.174(0.089)mtfemale_{ijkl} + -0.583(0.119)stblack_{ijkl}$$

$$\beta_{0ijkl} = 50.734(0.558) + f_{0l} + v_{0kl} + u_{0ijkl} + e_{0ijkl}$$

$$\beta_{1ijkl} = 50.777(0.528) + f_{1l} + v_{1kl} + u_{1ijkl} + e_{1ijkl}$$

$$\beta_{2kl} = 5.100(0.132) + f_{2l} + v_{2kl}$$

$$\beta_{3kl} = 3.109(0.080) + f_{3l} + v_{3kl}$$

$$\begin{bmatrix} f_{0l} \\ f_{1l} \\ f_{2l} \\ f_{3l} \end{bmatrix} \sim N(0, \Omega_f) : \Omega_f = \begin{bmatrix} 14.915(3.178) & & & \\ 12.232(2.797) & 13.062(2.839) & & \\ 2.071(0.609) & 2.135(0.595) & 0.736(0.177) & \\ 0.793(0.341) & 0.641(0.314) & 0.345(0.094) & 0.223(0.064) \end{bmatrix}$$

$$\begin{bmatrix} v_{0kl} \\ v_{1kl} \\ v_{2kl} \\ v_{3kl} \end{bmatrix} \sim N(0, \Omega_v) : \Omega_v = \begin{bmatrix} 55.557(1.836) & & & \\ 45.541(1.628) & 63.915(2.027) & & \\ 9.588(0.467) & 10.294(0.510) & 5.152(0.216) & \\ 4.519(0.374) & 3.752(0.389) & 2.014(0.133) & 2.513(0.137) \end{bmatrix}$$

$$\begin{bmatrix} u_{0ijkl} \\ u_{1ijkl} \end{bmatrix} \sim N(0, \Omega_u) : \Omega_u = \begin{bmatrix} 3.597(0.425) & \\ 5.489(0.292) & 13.970(0.388) \end{bmatrix}$$

$$\begin{bmatrix} e_{0ijkl} \\ e_{1ijkl} \end{bmatrix} \sim N(0, \Omega_e) : \Omega_e = \begin{bmatrix} 81.317(0.589) & \\ 0.000(0.000) & 26.556(0.236) \end{bmatrix}$$

*-2*loglikelihood(IGLS)* = 630207.200(88663 of 88663 cases in use)

Figure 4 Coefficient estimation for the between-student model from MLwin

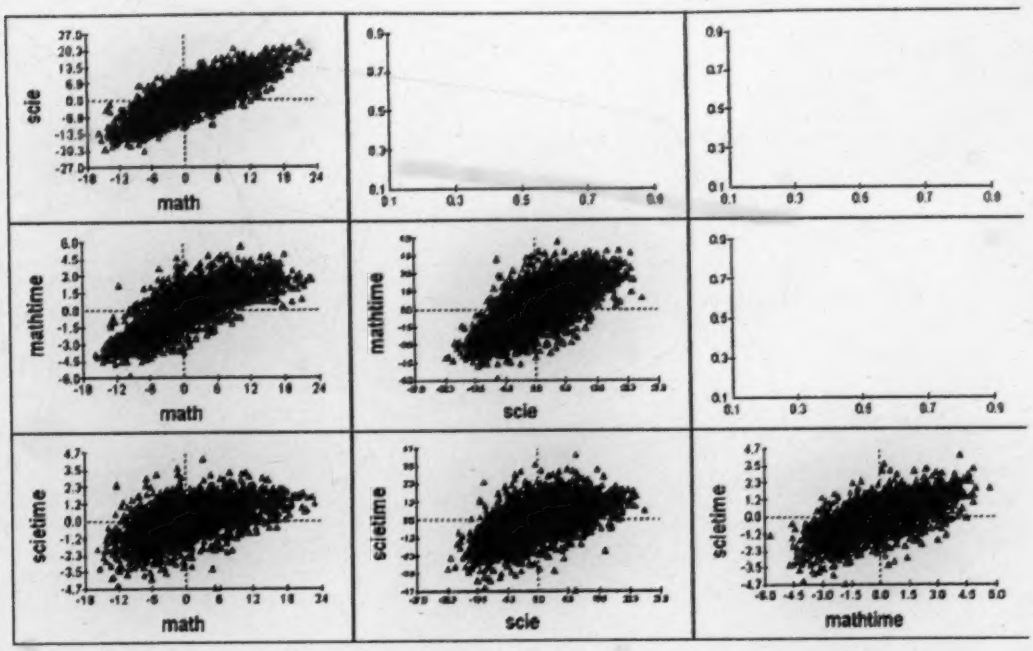

Figure 5. Scatter plot of the residuals at student level in the between-student model

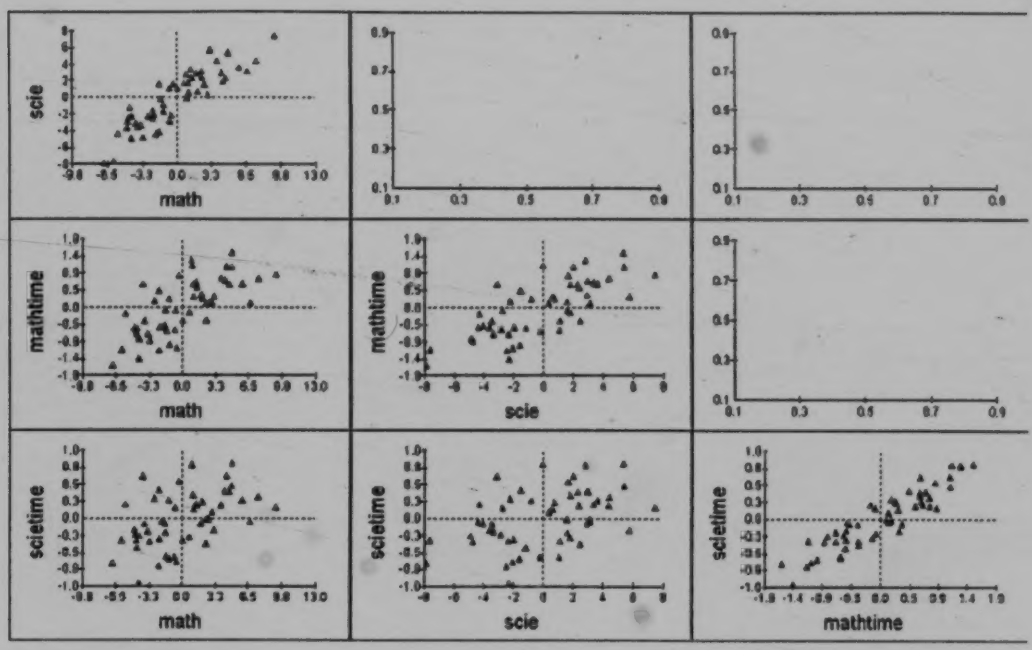

Figure 6. Scatter plot of the residuals at school level in the between-student model

$$score_{ijkl} \sim N(XB, \Omega)$$

$$score_{ijkl} = \beta_{0ijkl}math_{ijkl} + \beta_{1ijkl}scie_{ijkl} + \beta_{2kl}mathtime_{ijkl} + \beta_{3kl}scietime_{ijkl} + -0.053(0.007)mtage_{ijkl} + 0.260(0.118)mtparents_{ijkl} + 0.639(0.215)mtasian_{ijkl} + 0.182(0.089)mtfemale_{ijkl} + -0.594(0.119)stblack_{ijkl} + 0.192(0.089)strural_{ijkl} + -0.006(0.002)stfreelun_{ijkl} + -7.665(3.129)mtcompmath_{ijkl} + 0.269(0.088)mtchcom_{ijkl}$$

$$\beta_{0ijkl} = 50.726(0.537) + f_{0l} + v_{0kl} + u_{0jkl} + e_{0ijkl}$$

$$\beta_{1ijkl} = 50.777(0.527) + f_{1l} + v_{1kl} + u_{1jkl} + e_{1ijkl}$$

$$\beta_{2kl} = 5.110(0.128) + f_{2l} + v_{2kl}$$

$$\beta_{3kl} = 3.101(0.073) + f_{3l} + v_{3kl}$$

$$\begin{bmatrix} f_{0l} \\ f_{1l} \\ f_{2l} \\ f_{3l} \end{bmatrix} \sim N(0, \Omega_f) : \Omega_f = \begin{bmatrix} 14.858(3.166) & & & \\ 12.191(2.788) & 13.029(2.832) & & \\ 2.156(0.598) & 2.092(0.576) & 0.676(0.165) & \\ 0.513(0.304) & 0.292(0.280) & 0.278(0.081) & 0.173(0.054) \end{bmatrix}$$

$$\begin{bmatrix} v_{0kl} \\ v_{1kl} \\ v_{2kl} \\ v_{3kl} \end{bmatrix} \sim N(0, \Omega_v) : \Omega_v = \begin{bmatrix} 55.574(1.836) & & & \\ 45.537(1.628) & 63.888(2.027) & & \\ 9.602(0.467) & 10.307(0.511) & 5.160(0.216) & \\ 4.533(0.373) & 3.762(0.389) & 2.013(0.133) & 2.509(0.137) \end{bmatrix}$$

$$\begin{bmatrix} u_{0jkl} \\ u_{1jkl} \end{bmatrix} \sim N(0, \Omega_u) : \Omega_u = \begin{bmatrix} 3.590(0.425) & \\ 5.483(0.292) & 13.978(0.388) \end{bmatrix}$$

$$\begin{bmatrix} e_{0ijkl} \\ e_{1ijkl} \end{bmatrix} \sim N(0, \Omega_e) : \Omega_e = \begin{bmatrix} 81.317(0.589) & \\ 0.000(0.000) & 26.556(0.236) \end{bmatrix}$$

*-2*loglikelihood(IGLS)* = 630187.900(88663 of 88663 cases in use)

Figure 7. Coefficient estimation for the between-school model from MLwin

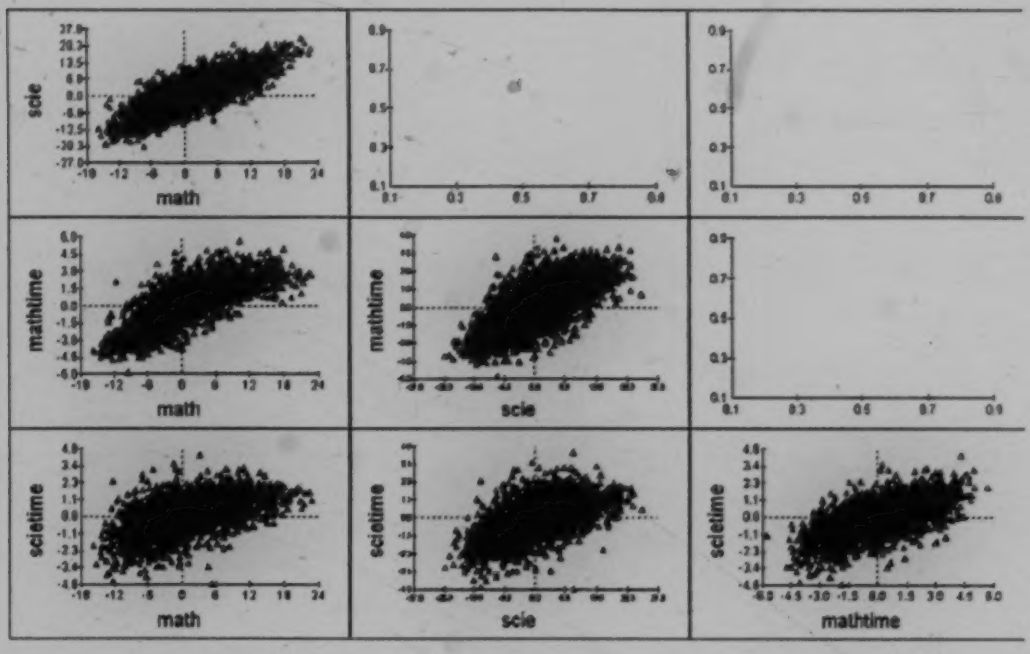

Figure 8. Scatter plot of the residuals at student level in the between-school model

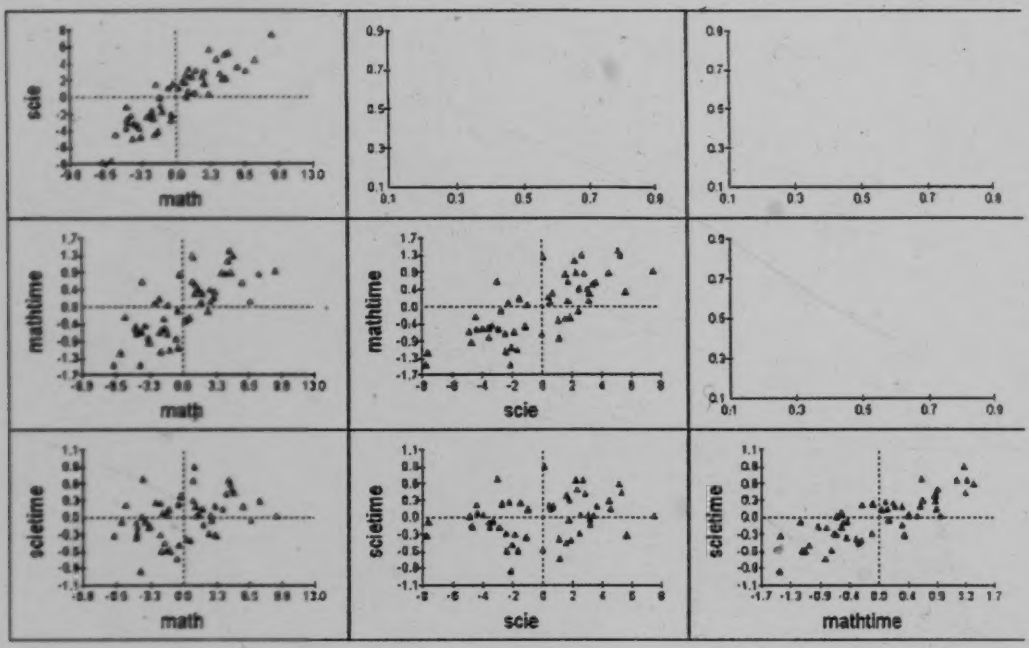

Figure 9. Scatter plot of the residuals at school level in the between-school model